Meddude Edexcel Discord Server

Proof

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2020 October Q1 P4

1. Given that n is an integer, use algebra, to prove by contradiction, that if n^3 is even then n is even.

(4)

2021 January Q3 P4

3. Prove by contradiction that there is no greatest odd integer.

(2)

2021 June Q9 P4

9. (i) Relative to a fixed origin O, the points A, B and C have position vectors **a**, **b** and **c** respectively.

Points A, B and C lie in a straight line, with B lying between A and C.

Given $AB:AC = 1:3$ show that

$$\mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$$

(3)

(ii) Given that $n \in \mathbb{N}$, prove by contradiction that if n^2 is a multiple of 3 then n is a multiple of 3

(5)

2021 October Q10 P4

10. (a) A student's attempt to answer the question

"Prove by contradiction that if n^3 is even, then n is even"

is shown below. Line 5 of the proof is missing.

Assume that there exists a number n such that n^3 is even, but n is odd.

If n is odd then $n = 2p + 1$ where $p \in \mathbb{Z}$

So $n^3 = (2p + 1)^3$

$= 8p^3 + 12p^2 + 6p + 1$

$=$

This contradicts our initial assumption, so if n^3 is even, then n is even.

Complete this proof by filling in line 5.

(1)

(b) Hence, prove by contradiction that $\sqrt[3]{2}$ is irrational.

(5)

2022 January Q6 P4

6. Three **consecutive** terms in a sequence of real numbers are given by

$$k, \ 1 + 2k \ \text{and} \ 3 + 3k$$

where k is a constant.

Use proof by contradiction to show that this sequence is not a geometric sequence.

(5)

2022 June Q9 P4

9. Use proof by contradiction to show that, when n is an integer,

$$n^2 - 2$$

is **never** divisible by 4

(4)

2022 October Q8 P4

8. A student was asked to prove by contradiction that

"there are no positive integers x and y such that $3x^2 + 2xy - y^2 = 25$"

The start of the student's proof is shown in the box below.

Assume that integers x and y exist such that $3x^2 + 2xy - y^2 = 25$

$$\Rightarrow (3x - y)(x + y) = 25$$

If $(3x - y) = 1$ and $(x + y) = 25$

$$\left.\begin{matrix} 3x - y = 1 \\ x + y = 25 \end{matrix}\right\} \Rightarrow 4x = 26 \Rightarrow x = 6.5, y = 18.5$$ Not integers

Show the calculations and statements that are needed to complete the proof.

(4)

2023 January Q9 P4

9. A student was asked to prove, for $p \in \mathbb{N}$, that

"if p^3 is a multiple of 3, then p must be a multiple of 3"

The start of the student's proof by contradiction is shown in the box below.

Assumption:

There exists a number p, $p \in \mathbb{N}$, such that p^3 is a multiple of 3, and p is NOT a multiple of 3

Let $p = 3k + 1$, $k \in \mathbb{N}$.

Consider $p^3 = (3k + 1)^3 = 27k^3 + 27k^2 + 9k + 1$

$= 3(9k^3 + 9k^2 + 3k) + 1$ which is not a multiple of 3

(a) Show the calculations and statements that are required to complete the proof.

(3)

(b) Hence prove, by contradiction, that $\sqrt[3]{3}$ is an irrational number.

(5)

2023 June Q7 P4

7. Use proof by contradiction to prove that $\sqrt{7}$ is irrational.

(You may assume that if k is an integer and k^2 is a multiple of 7 then k is a multiple of 7)

(4)

2023 October Q4 P4

4. (a) Prove by contradiction that for all positive numbers k

$$k + \frac{9}{k} \geqslant 6$$

(4)

(b) Show that the result in part (a) is not true for all real numbers.

(1)

2024 January Q8 P4

8. Use proof by contradiction to prove that the curve with equation

$$y = 2x + x^3 + \cos x$$

has no stationary points.

(4)